Asymptotic Behavior of Path Functionals for Vector-Valued Gaussian Processes at High Levels

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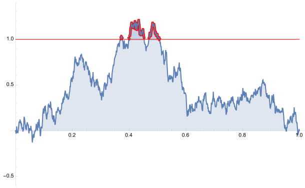
Extremes of Gaussian fields

Extreme value theory studies



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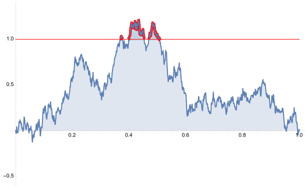

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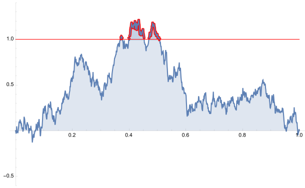

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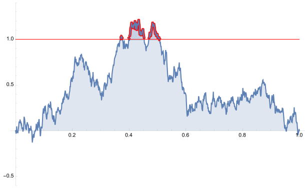

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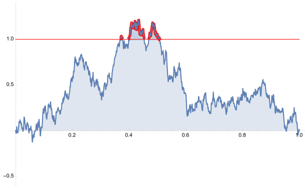

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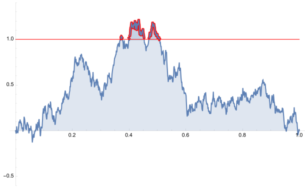

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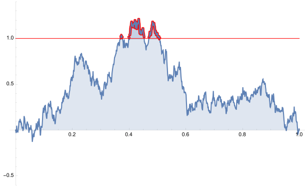

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Parisian functional

First, we want to include the so-called **Parisian functional** or **moving window infimum**

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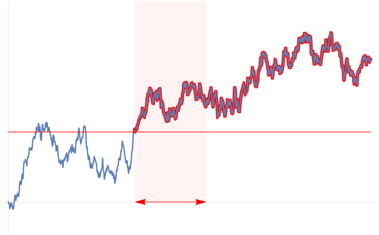
here's how the corresponding exceedance event looks:



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Area under the curve

One particular instance of the G-sojourn functional is the **area under the curve** functional

$$\Gamma_E(\boldsymbol{X}) = \int_E \min_{i=1,\dots,d} (X_i(t))_+ \, dt.$$

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this will be the *guiding principle* for our assumptions.

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$$\mathbb{P}\{\Gamma_{u^{-2/\alpha}S[k,k+1]}(\hat{\boldsymbol{u}}(\boldsymbol{X} - u\boldsymbol{b})) > L_u\} \sim H_\Gamma(S)\,\mathbb{P}\{\boldsymbol{X}(0) > u\boldsymbol{b}\}.$$

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The extension of this lemma to our " Γ " exceedance case is straightforward: just **swap** " $\exists\, t$ " by $\Gamma_{[0,T]}$.

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The key assumption that we need is

$$\Gamma(\boldsymbol{X}) > 0 \implies \exists\, t \in [0, T] : \boldsymbol{X}_t > \mathbf{0}. \tag{F1}$$

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If, futhermore, Γ satisfies

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then the **Pickands lemma** holds *exactly* as stated above:

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To this end, we impose some *global* (in time) assumptions on a **family of functionals**

$$\{\Gamma(A) : A \subset [0,T] \text{ compact}\}.$$

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we prove the following result:

Main theorem for stationary processes

Theorem 1.
If $\boldsymbol{X}$ satisfies R1[2] *and R2,* Γ *satisfies B1, B2, B3, F2 and F3 and* $L_u = L \cdot u^{-2\lambda/\alpha}$*, then*

$$\psi_{\Gamma, L_u}(u) \sim T\,\mathcal{H}\,u^{2/\alpha}\,\mathbb{P}\{\boldsymbol{X}(0) > u\boldsymbol{b}\}$$

with some complicated constant $\mathcal{H} \in (0, \infty)$.

[2] technical non-degeneracy assumption

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(D3) technical regularity assumption on the paths.

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$$g(t) = \min_{\boldsymbol{x} \geq \boldsymbol{b}} \boldsymbol{x}^\top R(t,t)^{-1} \boldsymbol{x}$$

attains its unique minimum at $t = 0$

(D2) $R(0,0) - R(t,s) \sim At^\beta + A^\top s^\beta + V|t-s|^\alpha$ as $t,s \to 0$ in $t \geq s \geq 0$

(D3) technical regularity assumption on the paths.

Under these assumptions we prove the following theorem.

Main theorem for non-stationary processes

Theorem 2.
Let $\boldsymbol{X}$ satisfy D1-D3, Γ satisfy B1, B2, B3, F2 and F3, then

$$\psi_{\Gamma,L_u}(u) \sim \mathcal{C}\, u^{(2/\alpha-2/\beta)_+}\, \mathbb{P}\{\boldsymbol{X}(0) > u\boldsymbol{b}\}$$

with some complicated constant $\mathcal{C} \in (0,\infty)$.