Let $S_t = \inf_{s\le t} X_s$.

By change of variables formula for Stiltjes integrals (note that the integral wrt dS_u is defined as the usual Riemann-Stiltjes integral, since S_u is a process of finite total variation (because it's monotone)) we have

$$\int_0^t \gamma(S_u)\,dS_u = \int_{S_0}^{S_t} \gamma(x)\,dx = G(S_t) - G(S_0), \quad \text{where} \quad G(t) = \int_0^t \gamma(x)\,dx.$$

Assume that our process X satisfies

$$S_0 = 0.$$

Then the exceedance event

$$\left\{\exists\, t \ge 0: \quad X_t - \int_0^t \gamma(S_r)\,dS_r > u\right\}$$

may be rewritten as

$$\{\exists\, t: \quad X_t - G(S_t) > u\}.$$

Assuming that $\gamma \ge 0$, we have that G is monotonically increasing, hence

$$G(S_t) = G\left(\inf_{0\le s\le t} X_s\right) = \inf_{0\le s\le t} G(X_s).$$

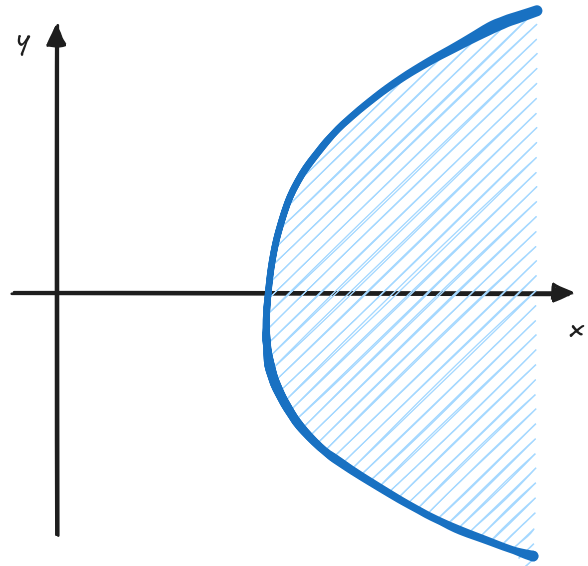
Therefore,

$$\begin{aligned}\left\{\exists\, t \ge 0: \quad X_t - \int_0^t \gamma(S_r)\,dS_r > u\right\} &= \{\exists\, t \ge 0,\ s\in[0,t]: \quad X_t - G(X_s) > u\} \\ &= \{\exists\, t \ge 0,\ s\in[0,t]: \quad \boldsymbol{X}(t,s) \in A_u\},\end{aligned}$$

where

$$\boldsymbol{X}(t,s) = \begin{pmatrix} X(t) \\ X(s) \end{pmatrix} \quad \text{and} \quad A_u = \{(x,y) : x - G(y) > u\}.$$

The set A_u looks something like this:



y
x